

MIDTERM EXAMINATION I

Directions: Do all three problems, which have unequal weight. This is a closed-book closed-note exam except for one $8\frac{1}{2} \times 11$ inch sheet containing any information you wish on both sides. A photocopy of the four inside covers of Griffiths is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (35 points)

A volume charge density $\rho(\vec{r})$ has the value over all space

$$\rho(\vec{r}) = \sigma_0 \delta(s - b) ,$$

where σ_0 and b are positive constants, and s is the usual cylindrical coordinate (the perpendicular distance from the z axis). (Remember that the usual integral formula for getting V from ρ can be used only if $\rho = 0$ at infinity!)

(a) (5 points) What are the dimensions of σ_0 ?

(b) (15 points) Find the electric field $\vec{E}(\vec{r})$ at any space point $\vec{r} = (s, \phi, z)$.

(c) (10 points) Find the potential difference

$$\Delta V \equiv V(b, 0, 0) - V(0, 0, \infty) ,$$

where, as above, the parentheses refer to (s, ϕ, z) .

Problem 2. (35 points)

The potential energy of mutual electrostatic interaction between two ideal electric dipoles \vec{p}_1 and \vec{p}_2 is

$$4\pi\epsilon_0 U_{12} = -p_1 p_2 \frac{3(\hat{r} \cdot \hat{p}_1)(\hat{r} \cdot \hat{p}_2) - \hat{p}_1 \cdot \hat{p}_2}{r^3} ,$$

where \vec{r} is their mutual separation. Consider a single ideal dipole with electric dipole moment equal in magnitude to p . The dipole is a distance z above an infinite grounded conducting plane $z = 0$.

(a) (10 points) Assume for this part that the dipole is allowed to rotate so that it may point in any direction. Does it tend to point perpendicular to the plane (*i.e.* along \hat{z}), or parallel to

the plane? Your answer should be justified by either a quantitative or a qualitative argument.

(b) (10 points) Assume instead that the dipole's direction is fixed so that it points perpendicular to the plane. Is the plane attracted to or repelled from the dipole? Again your answer should be justified by either a quantitative or a qualitative argument.

(c) (15 points) For the conditions of part (b), calculate the magnitude of this attractive or repulsive force.

Problem 3. (30 points)

A conducting sphere of radius b , centered at the origin, is surrounded by a spherical insulating layer of material with dielectric constant $\epsilon_r \equiv \epsilon/\epsilon_0$, extending from $b < r < 2b$. The conductor has a spherical hole of radius $b/4$ centered at $(x, y, z) = (0, 0, b/4)$. At the center of the hole is a charge q . There is no other *net* charge on any material. Take θ to be the usual spherical polar angle between \hat{z} and \hat{r} .

(a) (10 points) Find the free charge density $\sigma_f(\theta)$ on the outside surface of the conducting sphere.

(b) (20 points) Find the potential $V(\theta)$ at $r = b$, assuming that $V = 0$ at $r = \infty$.